

# “Combinatorics”

## Problem Set 4

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Class homepage:  
<http://carsten.codimi.de/comb07/>

### 4. Graphs

15. Classify all 3-regular graphs on 8 vertices up to isomorphism.
16. Let  $G$  be a  $k$ -connected graph,  $A, B \subset V$  disjoint sets of vertices, and  $|A| = |B| = k$ .
  - (i) Show that there are  $k$  vertex disjoint paths in  $G$ , each leading from a vertex in  $A$  to a vertex in  $B$ .
  - (ii) Let  $A = \{a_1, \dots, a_k\}$ ,  $B = \{b_1, \dots, b_k\}$ . Is it always true that there are vertex disjoint paths  $p_1, \dots, p_k$  with  $p_i$  leading from  $a_i$  to  $b_i$  for all  $i$ ?
17.
  - (i) Let  $G$  be a maximal plane bipartite graph with no vertex adjacent to all others. Show that  $G$  has only quadrilateral faces.
  - (ii) Show that any planar bipartite graph on  $n > 2$  vertices has at most  $2n - 4$  edges.
  - (iii) Show that  $K_{3,3}$  is not planar.
18. Show that every finite planar graph contains a triangle, or a vertex of degree at most 3, or both.