"Combinatorics"
Problem Set 5

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Version date: May 14, 2007
Issue date: May 15, 2007
Hand in date: May 23/24, 2007

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5. Finite Sets

19. Let $n \geq 0$. Show that the number of sequences $(a_1, \ldots, a_n)$ with $a_i \in \{+1, -1\}$ for all i and $\sum_{k=1}^{i} a_k \geq 0$ for all i equals $\binom{n}{\lfloor \frac{n}{2} \rfloor}$.

    Hint: DeBruijn, Tengbergen, Kruyswijk.

20. Let F be an antichain in $L_n(q)$ and let f_k denote the number of k-dimensional elements of F. Show that the LYM inequality

$$\sum_{k=0}^{n} \frac{f_k}{\begin{bmatrix} n \\ k \end{bmatrix}_q} \leq 1$$

    holds.

    Hint: $\begin{bmatrix} n \\ k \end{bmatrix}_q$ is the number of k-dimensional subspaces of an n-dimensional vector space over a field with q elements, this can be used without proof. It will not be necessary to know the number of maximal chains in $L_n(q)$.

21. Let $k \geq 1$ and $0 < m < \binom{2k-1}{k}$. Show that $\partial_k(m) > m$.

22. Let $0 \leq k \leq l \leq n$, $f_i \geq 0$ for $k \leq i \leq l$, and $f_k, f_l \neq 0$. Show that there is an antichain F in B_n with

$$\#\{M \in F \colon |M| = i\} = \begin{cases} 0, & l < i \leq n \\ f_i, & k \leq i \leq l \\ 0, & 0 \leq i < k \end{cases}$$

    if and only if

$$\partial_{k+1}(\cdots(\partial_{l-1}(\partial_l f_l + f_{l-1}) + f_{l-2}) \cdots + f_{k+1}) + f_k \leq \binom{n}{k}.$$