

“Combinatorics”

Problem Set 7

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Version date: June 5, 2007
Issue date: June 5, 2007
Hand in date: June 13/14, 2007

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7. Matroids

27. Let E be a set and \mathcal{I} the set of independent sets of a matroid of rank r on E .

(i) Let $0 \leq k \leq r$ and

$$\mathcal{I}' := \{A \in \mathcal{I} : |A| \leq k\}.$$

Show that (E, \mathcal{I}') is also a matroid. What is its rank?

(ii) Let $r \leq k \leq |E|$ and

$$\mathcal{I}'' := \{A \subseteq E : \text{there exists } I \in \mathcal{I} \text{ with } I \subseteq A \text{ and } |A \setminus I| \leq k - r\}.$$

Show that (E, \mathcal{I}'') is also a matroid. What is its rank?

28. Show that F_7 is not representable over a field of characteristic $\neq 2$ and that F_7^- is not representable over a field of characteristic 2.

29. Find representations over a field of your choice of $M(K_5)$ and of $M(K_5)^*$.

30. Describe the circuits of $M(K_5)^*$ and show that $M(K_5)^*$ is not graphic.