4. PROBLEM SET FOR "DIFFERENTIAL GEOMETRY II" AKA "ANALYSIS AND GEOMETRY ON MANIFOLDS" WINTER TERM 2009/10

Problem 10. Show that
$$\theta_t(x, y) = \left(x \cos t + 2y \sin t, -\frac{x}{2} \sin t + y \cos t \right)$$
defines an $\mathbb{R}$-action on $\mathbb{R}^2$ and determine its infinitesimal generator. What are the orbits of this action?

Problem 11. We consider the vector field
$$X = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$$
on $\mathbb{R}^2$. Is there an $\mathbb{R}$-action θ for which this is the corresponding infinitesimal generator? If so, then find it.

Problem 12. We consider the vector field
$$X = x^2 \frac{\partial}{\partial x}$$
on $\mathbb{R}$. Find the (maximal) local flow with this infinitesimal generator.