5. PROBLEM SET FOR "DIFFERENTIAL GEOMETRY II" AKA "ANALYSIS AND GEOMETRY ON MANIFOLDS" WINTER TERM 2009/10

Problem 13. Let M be a manifold, $X, Y \in \mathfrak{X}(M)$ and $f, g \in C^\infty(M)$. Prove that

$$[fX, gY] = fg[X, Y] + f(Xg)Y - g(Yf)X.$$

Problem 14. Given the vector fields in $\mathbb{R}^3$ (with coordinates x, y, z)

$$X = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}, \qquad Y = y\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}, \qquad Z = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z},$$

calculate $[X, Y]$, $[X, Z]$, $[Y, Z]$.

Problem 15. We consider $X, Y \in \mathfrak{X}(\mathbb{R}^2)$,

$$X = y\frac{\partial}{\partial x}, \qquad\qquad Y = \frac{\partial}{\partial y}.$$

Find the flow with infinitesimal generator X and use it to calculate $L_X Y$ from the definition. Compare with $[X, Y]$ calculated directly. Do the same with the roles of X and Y exchanged.

Hand-in date: November 18.