

**11. PROBLEM SET FOR “DIFFERENTIAL GEOMETRY II”  
AKA “ANALYSIS AND GEOMETRY ON MANIFOLDS”  
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**Problem 32.** Using the identity as a chart, we make

$$M = \{(x^1, x^2) \in \mathbb{R}^2 : x^2 > 0\}$$

into a 2-dimensional Riemannian manifold with the Riemannian metric given by  $g_{ij} = (x^2)^{-2}\delta_{ij}$ . Determine the values of  $\Gamma_{ij}^k$  and  $\Gamma_{ijk}$ .

**Problem 33** (continues Problem 32). For the 2-manifold  $M$  and still using the same coordinates, determine the values of  $R_{ijkl}$  and  $R_{ikl}^j$ . Also compute the sectional curvature (here called the Gaussian curvature, since  $M$  is 2-dimensional) at each point of  $M$ .

**Problem 34.** For an arbitrary Riemannian manifold and a local coordinate system, compute  $R_{ikl}^j$  and  $R_{ijkl}$  in terms of  $\Gamma_{ij}^k$  and  $g_{ij}$ .