

Topology

Problem Set 10

Prof. Günter M. Ziegler
Dr. Carsten Schultz

Version date: January 16, 2008
Issue date: January 16, 2008
Hand in date: January 23/24, 2008

Class homepage:
<http://carsten.codimi.de/top0708/>

10. Manifolds

34. Let $\alpha, \beta: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the following homeomorphisms.

$$\alpha(x, y) = (-y + 1, -x), \quad \beta(x, y) = (-y - 1, -x).$$

Let Γ be the subgroup of the group of homeomorphisms of \mathbb{R}^2 which is generated by these two elements.

- (i) Find an abelian subgroup of index 2 of Γ .
 - (ii) Find all relations between α and β .
 - (iii) Identify the orbit space \mathbb{R}^2/Γ .
 - (iv) The subgroup found in (i) corresponds to a covering of \mathbb{R}^2/Γ . Describe it.
35. Prove that $SO(3) \cong \mathbb{R}P^3$.
36. We can identify $O(k) \times O(l)$ with a subgroup of $O(k+l)$ by placing matrices diagonally next to each other. The set of cosets $O(k+l)/(O(k) \times O(l))$ can then be given the quotient topology. With these definitions, prove that $O(n+1)/(O(1) \times O(n)) \cong \mathbb{R}P^n$.
37. Show that the inclusion map $O(n) \rightarrow GL(n)$ is a homotopy equivalence.
Hint. Parametrize Gram-Schmidt.