"Topology"
Problem Set 3

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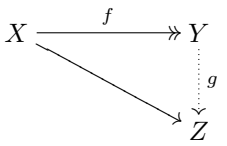
Class homepage:
`http://carsten.codimi.de/top0708/`

3. Homotopy

10. Prove the following useful lemma:

    Let X be a compact space, Y a Hausdorff space and $f\colon X \to Y$ a continuous surjection. Then for any space Z and any function $g\colon Y \to Z$ such that $g \circ f$ is continuous, the function g is continuous.



11. Let X be a contractible space and Y a *retract* of X, i.e. a subspace of X such that there exists a map $r\colon X \to Y$ sucht that $r|_Y = \mathrm{id}_Y$. Prove that Y is contractible.

12. Prove Lemma 3.9: If X is a k-connected space and $X \simeq Y$, then Y is k-connected.

13. Show that the following two subspaces of $\mathbb{R}^3$ are homotopy equivalent.

$$X := \left\{x \in \mathbb{R}^3 \colon \|x\| = 1\right\} \cup \left\{(x_1, x_2, 0) \colon x \in \mathbb{R}^2,\, \|x\| \leq 1\right\},$$
$$Y := \left\{x \in \mathbb{R}^3 \colon \|x\| = 1\right\} \cup \left\{x \in \mathbb{R}^3 \colon \|x - (0,0,2)\| = 1\right\}.$$

    You do not have to give explicit formulas and check continuity of all involved maps in detail if you give good descriptions of the homotopy equivalences and the involved homotopies in words and drawings.