

Topology

Problem Set 9

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9. Manifolds

31. (i) Prove (for example inductively, using Seiffert–van Kampen) that the fundamental group of a sphere with g crosscaps added and one point removed is a free group on g generators a_1, \dots, a_g , and that the a loop around the removed point represents the element $a_1^2 a_2^2 \cdots a_g^2$.
- (ii) Derive the presentation of $\pi_1(M'_g)$ given in Corollary 6.5 from this.
32. Describe explicitly a twofold covering of a Klein bottle by
- (i) a Klein bottle,
 - (ii) a torus.

What is the image of the fundamental group of the covering space in the fundamental group of the Klein bottle for the two coverings that you have created?

33. The connected sum $M \# N$ of two triangulated n -manifolds M and N without boundary is defined as follows: Remove an n -simplex from each of the manifolds, yielding two manifolds with boundary. Then glue these two together along a simplicial homeomorphism of their boundaries. (There is a choice here of the orientation of that homeomorphism, but the following questions are unaffected by it.)

How does $\chi(M \# N)$ depend on $\chi(M)$ and $\chi(N)$? Use this to prove that adding a crosscap to a surface decreases the Euler characteristic by one, while attaching a handle reduces it by 2.